# How Do You Know You’re Right If You Don’t Know What The Right Answer Is?

**Objective**: To understand the nature of scientific measurement and the fact that it always involves some uncertainty in the value. Learn how to quantify uncertainty and how to incorporate it in drawing conclusions based on measurements.

**Course Outcomes Addressed:**

1. Effectively collect, analyze and present data and correctly construct and interpret charts, graphs and tables to draw scientific conclusions;
2. Distinguish scientific studies from popular opinions by employing critical thinking skills and the scientific method;

**Requirements**: Stellarium planetarium simulator and Excel

**Strategy:** Put yourself in the role of the ancient Greek philosopher Eratosthenes and measure the size of the Earth (and also show that it is not flat).

The first person to make a credible attempt to determine the dimensions of the universe was an ancient Greek astronomer named Aristarchus. He seems to have been mostly interested in illustrating some techniques in geometry and didn’t try to make any precise measurements. However, he did show that the Sun is much further away and much larger than the Moon or Earth, and so it is probably no accident that he was one of the few ancient philosophers who believed in a Sun-centered rather than Earth-centered system.

Aristarchus derived a set of ratios between sizes and distances (e.g how far away is the Moon compared to the size of the Earth?). With a value for a single one of these quantities, he could have calculated all the others and gotten exact rather than relative sizes. But he didn’t have values for any of them.

The only one of Aristarchus’ numbers that would be plausible to measure at the time is the size of the Earth. But no one knew how to do it until, a few centuries later, a traveler told Eratosthenes an odd fact: in the city of Syene (modern day Aswan), there is a well and on the summer solstice the Sun shines straight down the well, creating no shadow.

From that, Eratosthenes knew he had it made. Well, almost. He worked in Alexandria, several hundred kilometers north of Syene. So he hired a bematist[[1]](#footnote-1) to walk from Alexandria to Syene and count how many steps it took.

## A. Why does the position of the Sun on the solstice matter? Eratosthenes’ Geometry

Having the distance to Syene, Eratosthenes stuck a pole in the ground at Alexandria on the Summer Solstice and measured the length of its shadow. The only reason it could have a shadow at all when Syene doesn’t is because the Earth curved between the two cities (something already well known from watching lunar eclipses).

1. Refer to Figure 2.10 in your text. Explain why, if the Sun is very far away, the rays of sunlight striking the Earth are almost exactly parallel to each other.

2. Refer to figure 2.11 in the text. Explain why the angle we want to measure is between the Sun and the point in the sky directly overhead (called the *zenith*), and why this measurement has to be made at local noon. Local noon = clock noon at the center of a time zone, but what would be true if you were a little east or a little west of the center?

3. Stellarium provides us with a coordinate system in the sky that will make this relatively easy for us, but Eratosthenes did not have that. All he had was his stick and its shadow. See if you can come up with a method that Eratosthenes could have used to zero in on local noon.

4. Explain why this equation has to be valid:

Here, *θ* is the angle from Syene to Alexandria. Think of it as the numb3r of degrees of latitude.

**Discuss your results with your instructor at this point**

## B. Collecting Data

***Setting up Stellarium for Measurements***

Enable the angle measuring tool Start up Stellarium. On the left side of the screen, there is a menu that pops out when you move your cursor on it. You’re looking for a wrench-and-star icon that will open the configuration window. Go to the Plugins tab and choose the Angle Measure item. At the bottom, there is a checkbox to load the angle measure tool at startup. Check that and restart Stellarium.

Change your Location Also on the left menu, at the top, is a compass rose that opens the location window. Eratosthenes was in Alexandria so let’s set our location to there. Type that in the search box and select the one in North Africa.

Set the date and time On the bottom of the screen is another menu that pops up when your cursor is over it. At the far right of that menu are some video controls. Hit pause. We don’t want the time to keep running while we make measurements. This will simulate simultaneous measurements. On the left menu just below the location tool is a clock. Click that to set the date and time. Eratosthenes performed his measurement on the Summer Solstice in 250 BCE so set the date to June 26, -239. Yes, the solstice was not on June 21 back then. Calendars drift over time due to precession of the equinoxes. Set your time to noon.

Dial in local noon On the bottom menu, toward the left, is an icon to turn on the azimuthal grid. That turns on a coordinate system in the sky. The lines in this grid converge at the zenith (the point directly overhead). There is a half circle from zero degrees to 180 degrees, passing through the zenith, called the *meridian*. At local noon, the Sun does NOT have to be at the zenith (in fact, from our latitude, the Sun is always a little bit south of the zenith). But is DOES have to be on the meridian. On the left menu, open the search window (a magnifying glass with a star). Search for the Sun.

1. You should see that the Sun is not quite on the meridian. Bring up the date/time window and slowly move time forward or backward until you get it there. Now you have found local noon.

2. Considering Figure 2.11 again, we want to measure the angle between two points on the screen. Which two points should we use?

3. Do the angle measurement. You do this by selecting the angle measure tool on the bottom menu, near the center. Then click on one point and drag to the other. Record the angle. Repeat this 10 times for each group member.

4. You also need the distance from Syene (aka Aswan) to Alexandria. Unlike Eratosthenes, you don’t have to pay someone to pace it off. You can use Google Maps. Zoom on Maps until you can see both cities. Right click on one of them. On the context menu that comes up, select measure distance. Then left click on the other city. A little box will pop up telling you the distance in both miles and kilometers. Do I need to say which you should use? Also repeat this 10 times for each group member.

5. Unless you enjoy spending about an hour of quality time with your calculator, Excel is your friend. We can tell it what to calculate and then to repeat that calculation on a range of data. You’ll be done in minutes. You’ll need to enter your angles and your distances in Excel. For the angles, it will be easiest if you put degrees, minutes and seconds in separate columns.

**Discuss your results with your instructor at this point**

## C. Analyzing the Data

Did you notice that for both angles and distances, you didn’t get the same number each time? That’s because every measurement has an uncontrollable element of randomness. Here, you don’t always put it exactly on the same point each time, even if you do your best. You can minimize it by being more careful but you can never eliminate it entirely. To properly interpret a measurement, it is just as important to quantify this uncertainty in a value as it is to get the value itself.

1. First of all, angles measured in degrees, minutes and seconds are impossible to work with. Thank the Babylonians for that. The first thing we need to do is convert the angles to decimal degrees. To do that, divide the seconds by 60 and add to the minutes. Then take that number, divide by 60, and add to the degrees. To make Excel do this, go to a new column and type an = sign. This tells Excel you are about to give it a formula to compute. Click the cell with the value for seconds, divide it by 60, type + and click on the cell with the value for minutes. Hit enter and you’ll see the result. You get something like this:

A screenshot of a spreadsheet


When you hit enter, you will see a small green box in the lower right corner of the cell.

A screenshot of a table


Left click that box and drag downwards to copy the calculation. You should end up with something like this:

A screenshot of a table

Description automatically generated

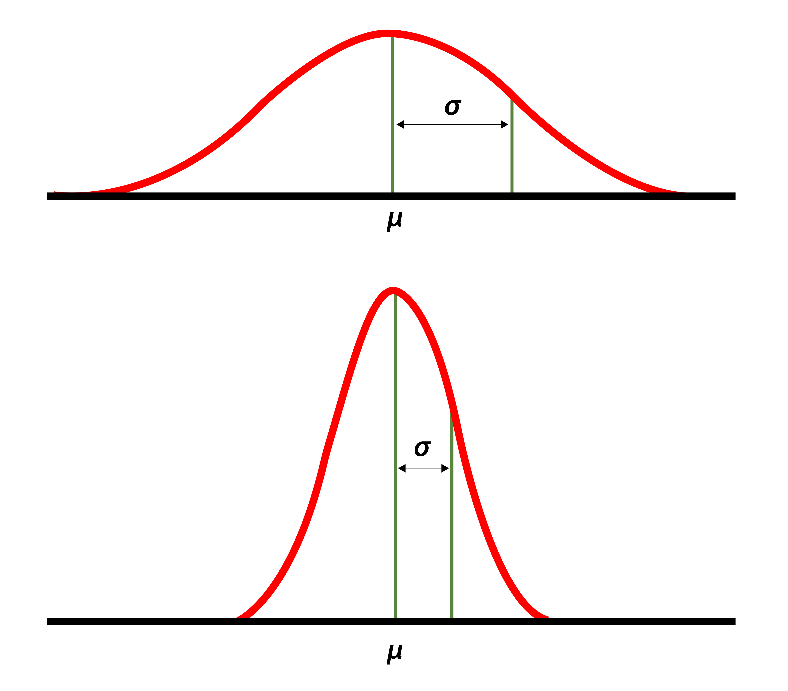
Now do the same thing to get decimal degrees: divide the decimal minutes by 60 and add to the degrees. You end up with:

A screenshot of a data table

Description automatically generated

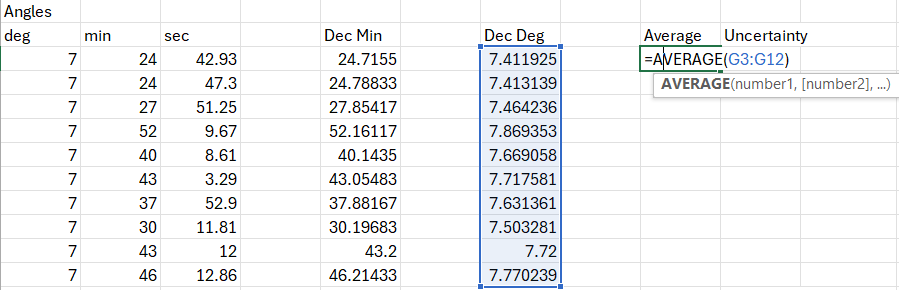
Magic! Fortunately, your distances are already decimal because the metric system is smarter than ancient Babylon. If you chose to use miles, I feel for you.

2. For both the decimal degrees and the distances, we are going to compute two things: the average value, which we will take as our best estimate, and the standard deviation, which we will use as an estimate of uncertainty. Standard deviation is essentially the average of how far each data value is from the average value. Typically, random variations are going to follow a bell shaped distribution and the standard deviation is a measure of how wide the bell is. In this picture, the top curve has more uncertainty than the bottom one.



Standard deviation is defined in such a way that 2/3 of the time you make an identical measurement, the result will lie between average – σ and average + σ. But that is only true if you make an infinite number of measurements so our results will only be an estimate. After all, if we could calculate the uncertainty, we’d compensate for it and have an exact value.

In Excel, the function you need for averaging a range of numbers is =AVERAGE(cell range). You can select the cell range by just clicking and dragging from the first one to the last one.



Do the same for standard deviation using =STDEV(cell range). You should do this for both the angles and the distances. You would report your answers as average ± standard deviation. That is, you are saying it is probably in the range average – standard deviation and average + standard deviation. NOTE: You would be hating life if you tried to use a calculator to do standard deviation.

**Discuss your results with your instructor at this point**

3. Now you’re ready to use your results to calculate the circumference of the Earth. Pick a cell to put the result in and write the formula for it using the average angle and distance.

4. We only have a single angle and a single distance so standard deviation won’t work to estimate the uncertainty in your circumference. Instead, we use a max/min method to estimate an upper and a lower bound on the value. For the upper bound, decide separately for the angle and the distance whether you want to use the + or the – to maximize the value of the circumference. Then do the same thing to minimize the value for a lower bound. You’ll end up with something like this:

A screenshot of a spreadsheet

Description automatically generated

5. Now assess your results. How close did your average come? Is the actual circumference between the upper and lower bounds?

6. Briefly, run through the same procedure in a new spreadsheet using the angle and distance measurements from all of your group members. What effect does it have on the average and the uncertainty?

**Discuss your results with your instructor at this point**

In this activity, we compared the calculated distance to the Sun to a known value. This is sort of the god’s eye view: “I know what value I am supposed to get so how far off am I?”

But this is not how science really works. You don’t know what value you are supposed to get. So how do you know you got the right value when you don’t know what the right value is? It is more productive to think of the variations in your measurements not as error but as uncertainty in the measurement due to aspects of it that are pretty random and uncontrollable (e.g. where exactly did you place your mouse cursor? How does zooming out affect your ability to make a precise measurement?)

If the uncertainty truly is random, you should overshoot the actual value as often as you undershoot it. So averaging the values should get you closer to the actual value than any individual one will. Similarly, how your values are scattered around the average gives you an estimate of the uncertainty in that value. Generally, in science, all you have to go on are the average and the uncertainty, one an estimate of the actual value and the other an estimate of how good that estimate is.

**Discuss your results with your instructor at this point**

## Summary

1. Suppose you are reading a paper that describes a measurement that you need for your own project. How would you assess the level of confidence you should have in the reported value?

2. Suppose you had two sources for that measurement that report different values. How would you decide whether they are likely to be actually the same value (with a little randomness) or actually different values?

3. Why do scientists typically want to make a large number of identical measurements instead of just a few?

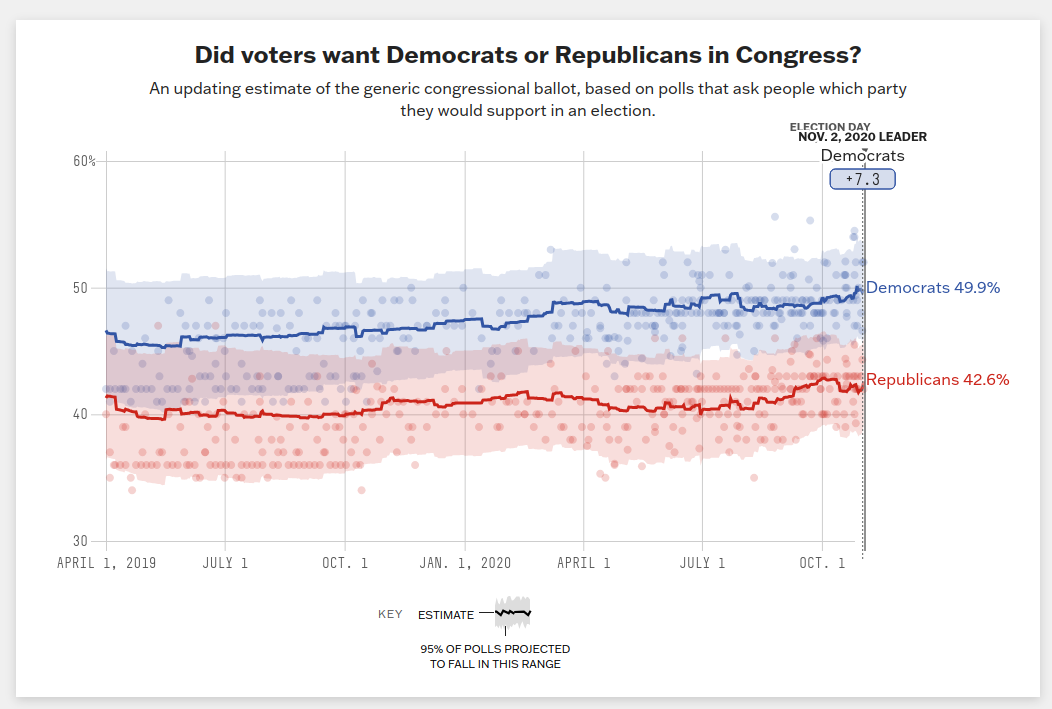
4. One of the assumptions built into using average and standard deviation is that random uncertainties in measurement give you a bell curve graph. The average value is the peak of the bell, which most measurements will be near with only a few further away, and the standard deviation is its width. Some people object to climate change because they don’t understand how a small change in the average global temperature can lead to such catastrophic events. Take a look at the animation “small change average” that you will find on D2L. See if you can use the idea of average and uncertainty to explain why a small change of the average temperature is important.

5. In the 2020 election, Ipsos/Reuters conducted a national poll just before election day that put Donald Trump at 45% and Joe Biden at 52%, each with an uncertainty of ± 3.7%. Given those results, could you assert complete confidence that Joe Biden would win the popular vote? Could you say he was likely to win? Explain your answer using the data.

6. This is a graph of a series of polls conducted by ABC called the Generic Congressional Ballot for the 2024 election and the 2020 election. The question is whether you would prefer Democrats or Republicans to control Congress. The solid lines are the average on each day and the colored bands are the uncertainty on each day.

A graph showing the results of the election

Description automatically generated



Estimate the uncertainty for each party using the graph. Could you have confidently asserted which party would control Congress in each of these elections? Explain. If you did not have the uncertainty available, just the averages, could you make a credible prediction?

**Discuss your results with your instructor at this point**

**When you have completed this activity, do the peer assessment for your group on D2L.**

1. Yes, this was an actual job. [↑](#footnote-ref-1)